1. The first layer of tape was stretched at the constant wrapping stress, σ_W, and "glued" to the rigid core, meaning that during deformation the radius of the first layer will not change.

$\sigma_\theta\,(r = r_o) = \sigma_W$;

2. The outer layer is a layer of tape that is stretched at the constant wrapping stress, σ_W, and remains in the same stretched state,

$\sigma_\theta\,(r = R) = \sigma_W$;

3. The outer surface of the tape has no additional normal or radial load (condition of free surface):

$\sigma_r\,(r = R) = 0$.

The first boundary condition corresponds to the cases of glued tape and a rigid core. Compared to experiments, this boundary condition produces a higher level of circumferential stresses in the first layer. The second boundary condition represents the case of a stretched and fixed end. In the case when the upper layer is released after wrapping, which is typical for experiments, circumferential stresses on the outer layer drop to zero.

Application of the three boundary conditions to the system of Equations 3.10 and 3.11 results in the following solutions for the unknown constants:

$$A = \frac{R^2\, r_o^2\, \sigma_W\, Ln\frac{r_o}{R}}{R^2 - r_o^2 - 2\, r_o^2 Ln\frac{r_o}{R}}; \qquad B = \frac{\left(R^2 - r_o^2\right)\sigma_W}{2\left(R^2 - r_o^2 - 2\, r_o^2 Ln\frac{r_o}{R}\right)};$$

$$C = -\sigma_W \frac{\left(R^2 - r_o^2\right)\left[1 + 2\, Ln(R)\right] + 2\, r_o^2 Ln\frac{r_o}{R}}{4\left(R^2 - r_o^2 - 2\, r_o^2 Ln\frac{r_o}{R}\right)} .$$

and subsequently the two stress components:

$$\sigma_\theta = \sigma_W \frac{\left(R^2 - r_o^2\right)\left(1 + Ln\frac{r}{R}\right) - \left(1 + \frac{R^2}{r^2}\right) r_o^2 Ln\frac{r_o}{R}}{R^2 - r_o^2 - 2\, r_o^2 Ln\frac{r_o}{R}}; \tag{3.12}$$

$$\sigma_r = \sigma_W \frac{\left(R^2 - r_o^2\right) Ln\frac{r}{R} - \left(1 - \frac{R^2}{r^2}\right) r_o^2\, Ln\frac{r_o}{R}}{R^2 - r_o^2 - 2\, r_o^2 Ln\frac{r_o}{R}}. \tag{3.13}$$

As can be seen from Equations 3.12 and 3.13, stresses in the roll of material are linearly proportional to the wrapping stress. These stresses are a combination of logarithmic functions with respect to the roll radius, a hyperbolic function of $1/r^2$, and constants representing the core radius and outer radius of the roll.

Equations 3.12 and 3.13 were then employed to compute the stress distribution in the roll of tape wrapped at a constant tensile stress (or draw load) around a rigid core. The example below includes the following parameters: core radius 120mm, outer radius of the roll 151.5mm, representing 10 wraps of a 3mm thick tape, and a constant wrapping stress of 1.38

x 10^5 kg/(mm s^2). The level of wrapping stress was chosen to be high in order to compare the solution with results of a finite element analysis. Young's modulus of the tape material is 1.637 x 10^6 kg/(mm s^2) and Poisson's ratio is 0.4.

Figures 5A-1, -2 and -3 show the distribution of circumferential and radial stresses along the radius of the roll. The amplitude of circumferential stress (Figure 5A-1) is higher than that of radial stress (Figure 5B-2). The distribution of circumferential stress along the roll radius resembles a skewed parabola with the left and right ends at the values of wrapping stress prescribed as boundary conditions. When the first wrap is "glued" to a softer core, a certain reduction in circumferential stress in the first layers would occur. In the middle of the roll, reduction in circumferential stress occurs due to a superposition of initial tensile stress and compression from upper layers resulting in the shrinkage (radius reduction) of the layers. Similarly, if the upper layer is not held in the stretched state but released, the stress on the right end of the curve would also reduce. As a result, both ends of the tape would have lesser stress than the initial value, and the ends of the parabolic curve would move down producing a shallower curve. This outcome would be desirable in the fabrication process as a smaller variation in the stress curves result in lesser variations in the strains and most important, in EFL.

As can be seen from Figure 5A-1, the "parabolic" circumferential stress curve for the 10-layer roll has depth of 0.72%, which is the difference between the maximum and minimum. For 50 layers, the depth of the circumferential stress parabola is 12.5%.

Comparison of the stress curves for 10 and 50 wraps indicated the following. For a smaller number of layers, the level of radial stress is smaller than that of circumferential stress. With the increase in the number of layers, the role of radial forces increases.